Sample Test Problems

1. Reduce the following rational expression to its lowest terms.

\[
\frac{12y^2 + 4y^2x^6}{8y^4x^6}
\]

2. Find the exact values of \(\tan\left(\frac{2\pi}{3}\right)\) and \(\csc\left(\frac{2\pi}{3}\right)\).

3. Rewrite \(\log_5 \frac{1}{125} = -3\) as an exponential equation.

4. Graph the following circle.

\[4x^2 + 4y^2 + 24x - 8y - 9 = 0\]

5. Suppose that the functions \(p\) and \(q\) are defined as follows.

\[p(x) = x^2 + 4\]
\[q(x) = \sqrt{x + 5}\]

Find \((q \circ p)(4)\) and \((p \circ q)(4)\).

6. Find all solutions of \(8\sin \theta + 4\sqrt{3} = 0\) in the interval \([0, 2\pi)\).

Write your answer(s) in radians in terms of \(\pi\).

7. Find the exact value of \(\tan\left(\cos^{-1}\left(-\frac{4}{5}\right)\right)\).

8. Solve for \(x\).

\[\log_2(x - 3) = 3 - \log_2(x - 10)\]

9. Use a sum or difference formula to find the exact value of the following.

\[\sin 18° \cos 27° + \cos 18° \sin 27°\]

10. Find the \(x\)-intercepts and \(y\)-intercepts of the graph of the function.

\[f(x) = -34x^3 - 21x^2 - 30x\]

11. Solve for \(x\).

\[2x^2 + 3y + 14 = (x - 2)^2\]
12. Answer the following.

   (a) Find an angle between $0^\circ$ and $360^\circ$ that is coterminal with $1025^\circ$.
   (b) Find an angle between 0 and $2\pi$ that is coterminal with $\frac{11\pi}{5}$.

13. Write the following in simplified radical form by rationalizing the denominator.

   \[
   \frac{5}{\sqrt{2}+\sqrt{6}}
   \]

14. The one-to-one function $g$ is defined below.

   \[g(x) = \frac{6x - 6}{-4x - 3}\]

   Find $g^{-1}(x)$, where $g^{-1}$ is the inverse of $g$.
   Also state the domain and range of $g^{-1}$ in interval notation.

15. Multiply.

   \[
   \frac{3x+6}{x-1} \cdot \frac{x^2+2x-3}{x^2-4}
   \]

16. Graph the solution to the following inequality on the number line.

   \[(x + 1)(x - 4) \geq 0\]

17. Find $\cos \theta$, $\sec \theta$, and $\cot \theta$, where $\theta$ is the angle shown in the figure. Give exact values, not decimal approximations.

18. Solve for $x$.

   \[64^{3x-1} = 8^{4x}\]

19. Solve for $u$, where $u$ is a real number.

   \[\sqrt{4u - 11} = u - 2\]

20. Solve for $x$.

   \[
   \frac{x+5}{x+2} - 1 = \frac{x-3}{x+5}
   \]